Summer Enrichment Packet for Rising Algebra 2 Students

Answer Key

PRINCE GEORGE'S COUNTY PUBLIC SCHOOLS
Division of Academics
Department of Curriculum and Instruction

BOARD OF EDUCATION
OF
PRINCE GEORGE'S COUNTY, MARYLAND

Note to Student: You've learned so much in Algebra I and Geometry! It is important that you keep practicing your mathematical knowledge over the summer to prepare yourself for Algebra II. In this packet, you will find weekly activities for the Summer Break.

Directions:

- Create a personal and fun math journal by stapling several pieces of paper together or use a notebook or binder with paper. Be creative and decorate the cover to show math in your world.

- Each journal entry should:
  - Have the week number and the problem number.
  - Have a clear and complete answer that explains your thinking.
  - Be neat and organized.

Playing board and card games are a good way to reinforce basic computation skills and mathematical reasoning. Try to play board and card games at least once a week. Some suggested games to play are: Monopoly, Chess, War, Battleship, Mancala, Dominoes, Phase 10, Yahtzee, 24 Challenge, Sudoku, Connect Four, and Risk.

Where to Go to Get Help and Opportunities for Practice!

During the course of your math work this summer, you may need some assistance with deepening your understanding the skills and concepts. You also might want to get some more practice. Here are some sites you can visit online:

COMMON CORE STATE STANDARDS INITIATIVE PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER	To get the exact definition of each standard, go to www.corestandards.org and search for the content standard (for example, *HSF.BF.1.3*).
KHAN ACADEMY	***Khan Academy*** has helpful videos and self-guided practice problems for every grade level. Go to www.khanacademy.org to get started.

Week 1

Domain: *Algebra*

Standards:
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Jesse's Phone Plan

Jesse has a prepaid phone plan (Plan A) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jesse uses only text, write an equation for the cost ***C*** of sending ***t*** texts.
C = 0.15t

a. How much will it cost Jesse to send 15 texts? Justify your answer.
C = 0.15t
= 0.15(15)
= 2.25
It will cost Jesse $2.25 to send 15 texts.

b. If Jesse has $6, how many texts can he send? Justify your answer.
C = 0.15t
6 = 0.15t
6/0.15 – 0.15t/0.15
40 = t
Jesse can send 40 texts with his $6.

2. If Jesse only uses the talking features of his plan, write an equation for the cost ***C*** of talking ***m*** minutes.
C = 0.10m

a. How much will it cost Jesse to talk for 15 minutes? Justify your answer.
C = 0.10m
= 0.10(15)
= 1.50
It will cost Jesse $1.50 for Jesse to talk for 15 minutes.

b. If Jesse has $6, how many minutes can he talk? Justify your answer.

C = 0.10m
6 = 0.10m
6/0.10 – 0.10m/0.10
60 = m
Jesse can talk for 60 minutes with his $6

3. If Jesse uses both talk and text, write an equation for the cost ***C*** of sending ***t*** texts and talking ***m*** minutes.

C = 0.15t + 0.10m

a. How much will it cost Jesse to send 7 texts and talk for 12 minutes? Justify your answer.

C = 0.15t + 0.10m
C = 0.15(7) + 0.10(12)
C = 1.05 + 1.20
C = 2.25

b. If Jesse wants to send 21 texts and only has $6, how many minutes can he talk?

C ≤ 0.15t + 0.10m
6 ≤ 0.15(21) + 0.10m
6≤ 3.15 + 0.10m
6 ≤ 3.15 = 0.10m
2.85 ≤ 0.10m
2.85/0.10 ≤ 0.10m/0.10
28.5 ≤ m

After sending 21 texts, Jesse can only talk up to 28 minutes.

c. Will this use all of his money? If not, will how much money will he have left? Justify your answer. **Jesse will not use all of his money. If he talks 28 minutes, his cost, including the 21 texts, will be $5.95. He will have a nickel left.**

C = 0.15(21) + 0.10(28)
C = 5.95

4. Jesse discovers another prepaid phone plan (Plan B) that charges a flat fee of $15 per month, then $.05 per text sent or minute used. Write an equation for the cost of Plan B.

C = 0.05t + 15

5. In an average month, Jesse sends 200 texts and talks for 100 minutes. Which plan will cost Jesse the least amount of money? Justify your answer.

Plan A

C = 0.15t + 0.10m

C = 0.15(200) + 0.10(100)

C = 30 + 10
C = 40

Plan B

C = 0.05t + 15
C = 0.05(200) + 15
C = 10 + 15
C = 25

Plan B will cost the least amount of money. Jesse will save $15 on his average use on this plan.

Week 2

Domain: *Algebra; Number and Quantity*

Standards:

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

IVY CARTER GROWS UP

You are a medical assistant in a pediatrician's office and one of your responsibilities is evaluating the growth of newborns and infants. Your first patient, a baby girl named Ivy Carter, was 21.5 inches long at 3 months old. At 8 months, you measure her at 24 inches long. For your medical records, all measurements must be given both in inches and in centimeters: 1 inch = 2.54 cm

1. Assuming Ivy's growth is linear, find a linear model for her growth (in inches) over time (in months).

Let x= time in months
Let y= length in inches

Two points are given: (3, 21.5) (8, 24)

To find an equation the student must first find slope. Students should know that slope (m) is rise over run or:

$$m = \frac{y1 - y2}{x1 - x2} = \frac{24 - 21.5}{8 - 3} = \frac{2.5}{5} = \frac{1}{2}$$

Using the slope in the y-intercept form for the line, we can use either pair of the above points to solve for the intercept.

Using (3, 21.5)	Or using (8, 24)
y = ½ x + b 21.5 = ½ (3) + b 21.5 = 1.5 + b 20 = b	y = ½ x + b 24 = ½ (8) + b 24 = 4 + b 20 = b

Ivy's growth is modeled by the equation: y = ½ x + 20

2. Use your model to determine how long Ivy was at birth (in centimeters)? Explain how you know your answer is correct, assuming this model.

Substitute 0 months for x in the equation to find the length at birth:
y = ½ (0) + 20
y = 20 inches

Ivy was 20 inches at birth. Using 1 inch = 2.54 centimeters:

20 inches x 2.54 cm/in = 50.8 centimeters

Using the attached growth chart we might also estimate the number of centimeters in 20 inches: from looking at the vertical scale, which is in inches and centimeters, we see that 20 inches corresponds with just over 50 cm. This validates our answer.

3. Use your model to determine approximately how tall Ivy will be at 1 year old. At 3 years old (in centimeters). Show how you know your answers are correct.

To find Ivy's height at 1 year, substitute 12 months into the equation for x:
y = ½ (12) + 20
y = 6 + 20
y = 26 inches

Ivy will be 26 inches at 1 year old (12 months).

26 in x 2.54 cm/in = **66.04 centimeters**

Ivy will be approximately 66 centimeters tall at 1 year old (12 months).

To find Ivy's height at 3 years, substitute 36 months into the equation for x:
y= ½ (36) + 20
y = 18 + 20
y = 38 inches

Ivy will be 38 inches at 3 years (36 months).

38 in x 2.54 cm/in = **96.52 centimeters**

Ivy will be almost 97 centimeters tall at 3 years old (36 months).

For both parts of this question, the vertical scale on the attached growth chart might also be used to estimate the conversion of inches to centimeters since the vertical scale is given in both units.

4. Use your model to estimate how old Ivy will be (in years and months) when she measures at 48 inches. Show how you know your estimate is accurate.

To find Ivy's age at 48 inches tall, substitute 48 inches into the equation for y:
$48 = \frac{1}{2}x + 20$
$28 = \frac{1}{2}x$
$x = 56$ months

Dividing 56 by 12 months per year: 56 / 12 = 4 years and 8 months

When Ivy is 48 inches tall she will be 4 years 8 months old.

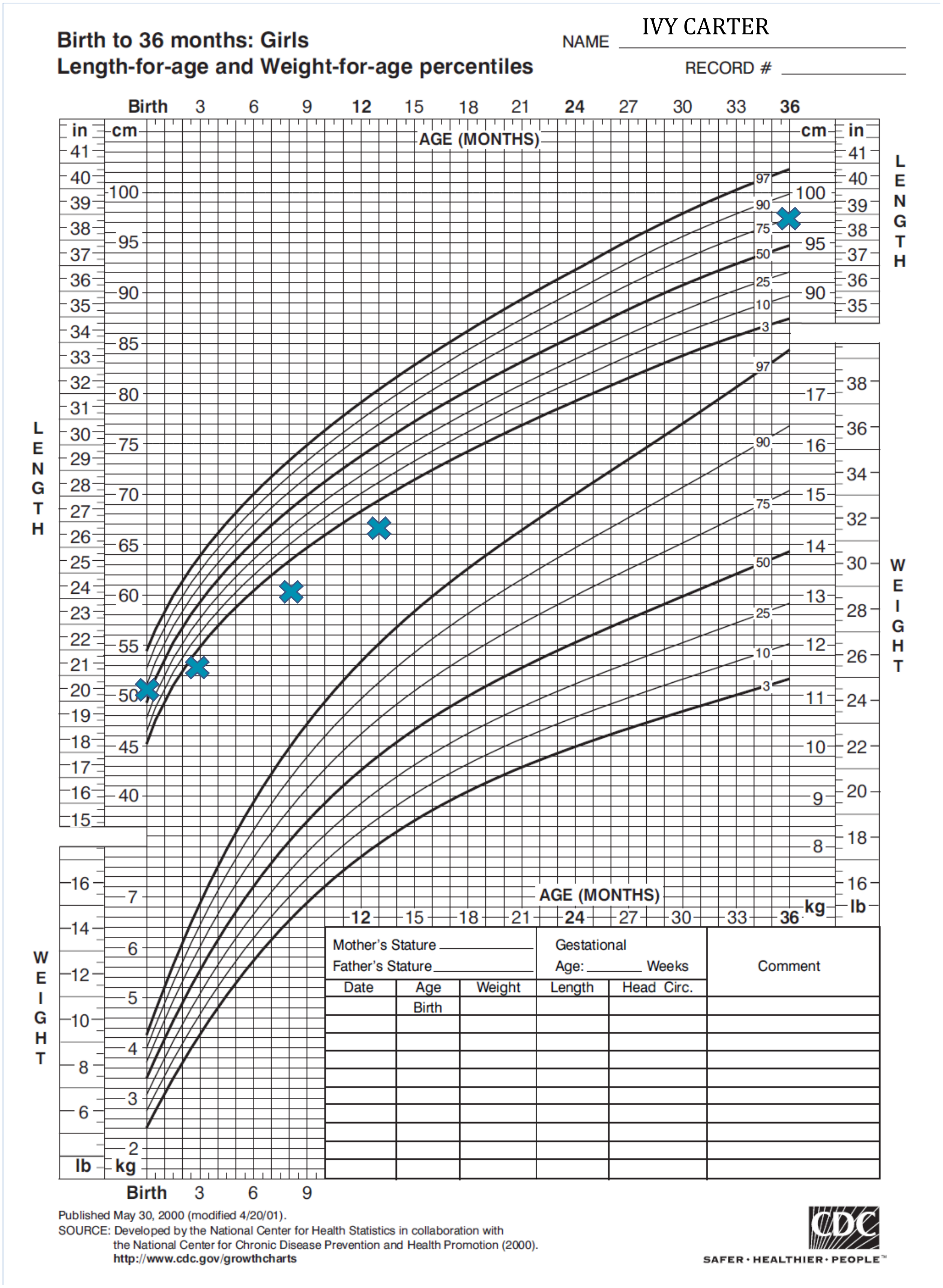
5. Complete the table below and us the chart to plot Ivy Carter's growth, based on the calculations above (record # 1234.56).[1]

Age (months)	Length (in)	Length (cm)
0	20	50.8
3	21.5	54.61
8	24	60.96
12	26	66.04
36	38	96.52
56	48	121.92

6. What is Ivy's approximate length-for-age percentile at each of these ages?

Age (months)	Length (in)	Length (cm)	Percentile
0	20	50.8	50th
3	21.5	54.61	3rd
8	24	60.96	< 3rd
12	26	66.04	< 3rd
36	38	96.52	< 75th
56	48	121.92	Not available

[1] http://www.cdc.gov/growthcharts/charts.htm#Set1


Birth to 36 months: Girls
Length-for-age and Weight-for-age percentiles
NAME IVY CARTER
RECORD #
AGE (MONTHS)
Birth 3 6 9 12 15 18 21 24 27 30 33 36
LENGTH
WEIGHT
in cm
kg lb
97 90 75 50 25 10 3
Mother's Stature
Father's Stature
Gestational Age: Weeks
Comment
Date Age Weight Length Head Circ.
Birth
Published May 30, 2000 (modified 4/20/01).
SOURCE: Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion (2000).
http://www.cdc.gov/growthcharts
SAFER · HEALTHIER · PEOPLE

Week 3
Domain: *Algebra; Number and Quantity*

Standards:
N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
A-CED.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A-REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7.RP3: Use proportional relationships to solve multistep ratio and percent problems.

DENTAL IMPRESSIONS

To fabricate a stone model from a dental impression, you need 30 mL of water to 100 grams of gypsum powder. In your orthodontic office, in an average week, you make 75 impressions. It is most cost effective to order the powder in 50-lb boxes. (Remember: 1 gram = .0022 pounds)

1. How many stone models can you make with one 50-lb box? Show your work and mathematical thinking.

Since 100 grams of gypsum powder are needed per impression, we can first determine how many 100-gram units are in 50 pounds:

$$\frac{100 \text{ grams}}{1 \text{ impression}} \times \frac{.0022 \text{ lbs}}{1 \text{ gram}} = 0.22 \text{ lbs per impression}$$

Then to find how many impressions per 50lb box:

$$\frac{50 \text{ lbs}}{1 \text{ box}} \times \frac{1 \text{ impression}}{0.22 \text{ lbs}} = 227.27 \text{ impressions per box}$$

OR if we first convert the 50-pound box into grams:

$$50 \text{ lbs} \times \frac{1 \text{ g}}{.0022 \text{ lbs}} = 22,727.27 \text{ g (in one 50-lb box)}$$

If we divide into 100-gram units we need for each impression, we get: 227.27 impressions per box

227 impressions are possible using one 50-lb box of gypsum powder

2. How frequently will you need to re-order your powder supply? Show your work and mathematical thinking.

We found that we can make 227 impressions per box and we know that the office creates 75 impressions per week. Using dimensional analysis we find that:

$$\frac{227 \text{ impressions}}{1 \text{ box}} \text{ x } \frac{1 \text{ week}}{75 \text{ impressions}} = 3.03 \text{ weeks per box}$$

The office would need to reorder once every 3 weeks, on average. This assumes prompt delivery of your order and assumes that you do not want to maintain excess boxes in your inventory.

3. What is your office's annual demand for the gypsum powder? Show your work and mathematical thinking.

There are 52 weeks in a year, and the office must order a box every 3 weeks, therefore the office must order: 52 / 3 = 17.3

The office should order 18 boxes per year.
(Students should round up to ensure the office doesn't ever run out).

4. Your office is considering purchasing digital impression technology, but only if it proves to be more cost effective within two years. Assuming the initial investment in technology and training for the digital impression scanner is $115,000, the technology will eliminate the number of manual/non-digital impressions by 60%, and manual impressions cost your office $30 on average, create equations to determine your break-even point on this investment. *Support your solution both algebraically and graphically.*

Let x = time, in years
Let y = cost, in dollars

The student must create two equations, one for the office without the technology and one for the office with the technology.

Without Technology:

The office makes 75 impressions a week for 52 weeks a year at a cost of $30.00 per impression. Therefore the annual cost per year of impressions is:

$$\frac{75 \text{ impressions}}{1 \text{ week}} \times \frac{52 \text{ weeks}}{1 \text{ year}} \times \frac{\$30}{1 \text{ impression}} = \$117{,}000 \text{ per year}$$

So the equation representing the cost per year without technology is:

$y = 117{,}000x$

With Technology:

The office still must make 40% of impressions manually (at the cost of $30 per impression). The office makes 75 impressions a week, at 52 weeks a year so the number of impressions per year is:

75 x 52 = 3900 impressions per year

The office now only needs to manually make 40% of this amount, therefore

40% of 3900 = 0.4 x 3900 = 1,560 impression will be made manually.

At $30 an impression the annual cost for manual impressions would now be:

1,560 x 30 = $46,800

Therefore, the equation will be the total cost of the technology plus $46,800 each year:

$y = 46{,}800x + 115{,}000$

To solve for the break-even point we can solve the system of equations:

$y = 117{,}000x$

$y = 46{,}800x + 115{,}000$

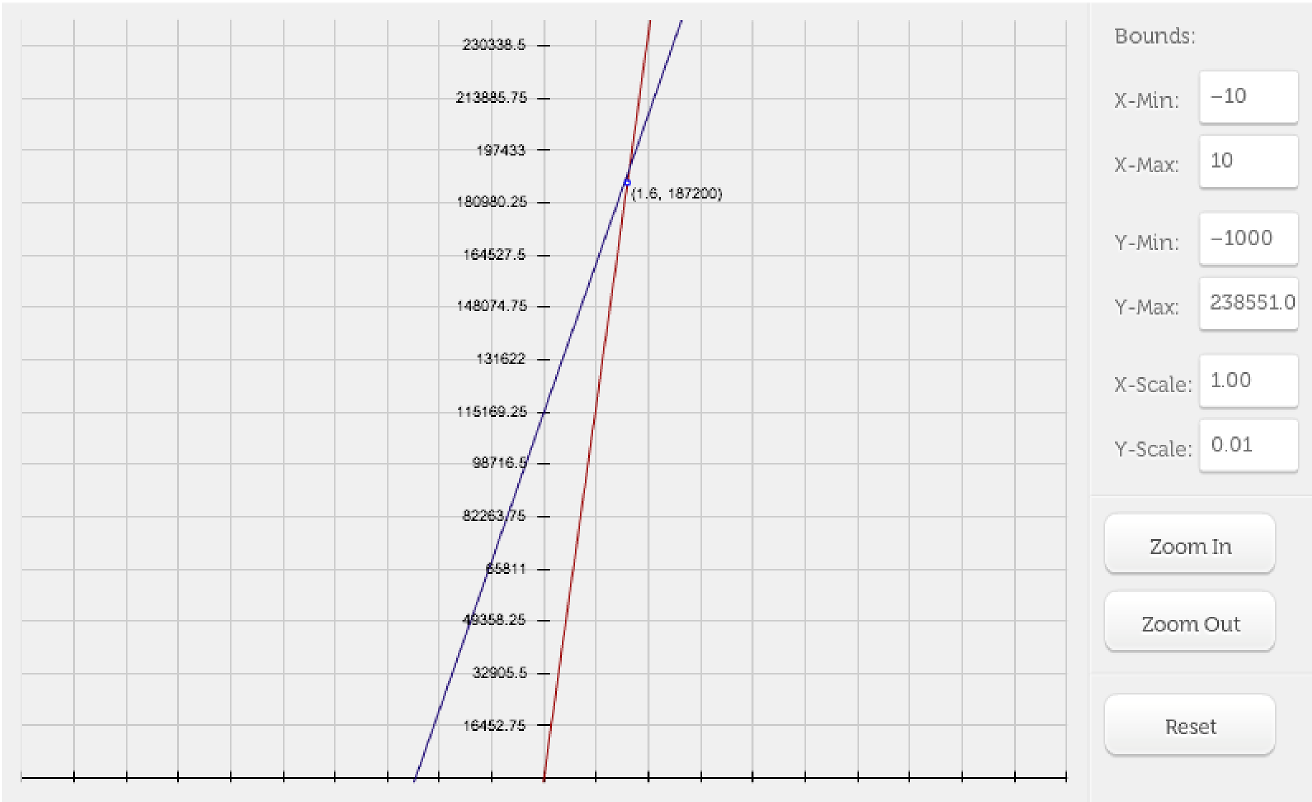
$117{,}000x = 46{,}800x + 115{,}000$

$70{,}200x = 115{,}000$

$x = 115{,}000/70{,}200 = 1.64$ years or approximately 1 year, 8 months

Therefore, the break-even point on this investment will be after 1.64 years, or approximately 1 year, 8 months.

The student must also solve graphically and show the intersection point. (This graph was created using the online graphing calculator tool: Meta-Calculator.)



5. Write a recommendation to your office encouraging or discouraging the purchase of the digital impression technology based on your findings above.

Answers may vary, but the investment in the digital impression technology pays off after 1.64 years, or in about 1 year and 8 months, making it cost effective within two years. It is, therefore, a cost effective decision for the office to purchase this technology if they are planning to use it for 2 years or more.

Week 4

Domain: *Algebra; Number and Quantity*

Standards:

A.REI.6 Solve systems of linear equations exactly and approximately focusing on pairs of linear equations in two variables.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*

Where's the Beef?

On average a grocer sells the following amounts of ground beef each week:

- 485 lbs of Regular Ground Beef (25% fat)
- 1010 lbs of Lean Ground Beef (18% fat)
- 537 lbs of Extra Lean Ground Beef (12% fat)

He orders boneless round and lean trim beef products from which he produces his own ground beef. He orders:

Beef Type	% Lean	Price per Pound
Boneless Round	95%	$2.13
Lean Trim	70%	$1.82

Fill in the following table as you answer questions 1 and 2 below:

Type	Regular Ground Beef (lbs)	Lean Ground Beef (lbs)	Extra Lean Ground (lbs)	TOTALS
Boneless Round	**97 lbs**	**484.8 lbs**	**386.6 lbs**	**968.4 lbs**
Lean Trim	**388 lbs**	**525.2 lbs**	**150.4lbs**	**1063.6 lbs**
TOTALS	**485 lbs**	**1010 lbs**	**537 lbs**	**2032 lbs**

Show <u>all</u> your work:

1. How many pounds of *boneless round* and *lean trim* are needed to make each of the ground beef mixtures to meet demand?

Method 1: Systems of Equations
Create a system of equations for each type of ground beef (ground beef, lean ground beef, and extra lean ground beef) based on the number of pounds sold each week, and the lean percentage contents of each mixture in order to determine how much boneless round and lean trim create each type of ground meat mixture.

Let x = Boneless Round
Let y = Lean Trim

Regular Ground Beef	**Lean Ground Beef**	**Extra Lean Ground Beef**
To solve the system by substitution: x + y = 485 0.95x + 0.70y = 0.75(485) = 363.75 Solve the first equation for x ∧ x = 485 – y. Substitute this into the second equation to solve: 0.95(485 – y) + 0.70y = 0.75(485) 460.75 - 0.95y + 0.70y = 363.75 - 0.25y = - 97 **y = <u>388 lbs of Lean Trim</u>** Then substitute this into the original equation to solve for x: x + 388 = 485 **x = <u>97 lbs of Boneless Round</u>**	To solve the system by substitution: x + y = 1010 0.95x + 0.70y = 0.82(1010) Solve the first equation for x ∧ x = 1010 – y. Substitute this into the second equation to solve: 0.95(1010 – y) + 0.70y = .82(1010) 959.50 - 0.95y + 0.70y = 828.2 0.25y = 131.3 **y = <u>525.2 lbs of Lean Trim</u>** Then substitute this into the original equation to solve for x: x + 525.2 = 1010 **x = <u>484.8 lbs of Boneless Round</u>**	To solve the system by substitution: x + y = 537 0.95x + .070y = .88(537) Solve the first equation to ∧ x = 537 – y. Substitute this into the second equation to solve: 0.95(537 – y) + 0.70y = .88(537) 510.15 - 0.95y + 0.70y = 472.56 0.25y = 37.59 **y = <u>150.4 lbs of Lean Trim</u>** Then substitute this into the original equation to solve for x: x + 150.36 = 537 **x = <u>386.6 lbs of Boneless Round</u>**

OR solve the system by addition/elimination. Here is an example using regular ground beef:

Multiply both sides of the first equation by -0.7 to get
- 0.7x – 0.7y = - 0.7(485) = - 339.5

Now add the new equation to the second equation to eliminate the y-variable:

0.95x + 0.70y = 363.75
<u>+ - 0.7x – 0.7y = - 339.5</u>

$0.25x = 24.25$

$x = 24.25 / 0.25$

$\mathbf{x = 97}$ **lbs,** Then find y using substitution: **y = 388 lbs**

2. How many total pounds of each type will he need to buy each week?

The grocer needs **968.4 lbs of Boneless Round and 1036.6 lbs of Lean Trim Beef**.

3. To make a 15% profit, how much does he have to charge per pound for each ground beef mixture?

$$\frac{Revenue(R) - Cost}{Cost} = \% \text{ Profit}$$

This equation shows the ratio of profit to cost, which for this task must be 15%. For each percentage lean ground beef the following equation can be used to determine the total cost:

Cost = (Pounds Boneless Round)($2.13 per pound) + (Pounds Lean Trim)($1.82 per pound) = 2.13x + 1.82y

Regular Ground Beef	Lean Ground Beef	Extra Lean Ground Beef
Cost = 97(2.13) + 388(1.82) **= $912.77** Now put this into the Percentage Profit Formula and solve for R: $\frac{R - 912.77}{912.77} = 15\%$ R – 912.77 = 0.15(912.77) R = 136.9155 + 912.77 = 1049.69 **Revenue = $1049.69** Divide this total revenue by the total number of pounds of ground beef (485) to find the cost per pound: 1049.69/485 = **$2.16 per pound for Regular Ground Beef.**	Cost = 484.8(2.13) + 525.2(1.82) **= $1988.48** Now put this into the Percentage Profit Formula and solve for R: $\frac{R - 1988.48}{1988.48} = 15\%$ R – 1988.48 = 0.15(1988.48) R = 298.272 + 1988.48 = 2286.75 **Revenue = $2286.75** Divide this total revenue by the total number of pounds (1010) to find the cost per pound: 2286.75/1010 = **$2.26 per pound for Lean Ground Beef.**	Cost = 386.6(2.13) + 150.4(1.82) **= $1097.19** Now put this into the Percentage Profit Formula and solve for R: $\frac{R - 1097.19}{1097.19} = 15\%$ R – 1097.19 = 0.15(1097.19) R = 164.5785 + 1097.19 = 1261.77 **Revenue = $1261.77** Divide this total revenue by the total number of pounds (537) to find the cost per pound: 1261.77/537 = **$2.35 per pound for Extra Lean Ground Beef.**

4. On which product is he making the higher profit and how much is that profit?

Profit = Revenue – Total Cost = (Cost per pound)(total pounds) – Total Cost

Ground Beef: $2.16(485) – $912.77 = $134.83
Lean Ground Beef: $2.26(1010) – $1988.48 = $294.12
Extra Lean Ground Beef: $2.35(537) – $1097.19 = $164.76

He makes the most profit with the Lean Ground Beef. He makes $294.12 per week.

Note: If the Revenue and Cost figures from the table above are used, profit figures may vary slightly due to rounding, but the largest profit is still earned from the Lean Ground Beef.

Ground Beef: $1049.69 - $912.77 = $136.92
Lean Ground Beef: $2286.75 - $1988.48 = $298.27
Extra Lean Ground Beef: $1261.77 - $1097.19 = $164.58

5. What is his total profit?

Total Profit: $134.83 + $294.12 + $164.76 = **<u>$593.71</u>**

<u>Total Profit using profit figures derived from the table result in a slightly different total profit due to rounding:</u>

<u>Total Profit: $136.92 + $298.27 + $164.58 = $599.77</u>

6. The grocer has the option to purchase Bull Meat, which is 92% lean and costs $2.07 per pound. If the grocer replaces the Boneless Round with the Bull Meat, how much will his total profit change, assuming that the charge per pound (calculated in question 3) remains the same?

Regular Ground Beef – Using Bull Meat	**Lean Ground Beef – Using Bull Meat**	**Extra Lean Ground Beef – Using Bull Meat**
x + y = 485 0.92x + 0.70y = 0.75(485) Solve the first equation for x: x = 485 – y. Substitute this into the second equation to solve: 0.92(485 – y) + 0.70y = 0.75(485) 446.2 – 0.92y + .70y = 363.75 0.22y = 82.45 y = **<u>374.8 lbs</u>** Substitute this into the original equation to solve for x: x = 485 – y x = **<u>110.2 lbs</u>**	x + y = 1010 0.92x + 0.70y = 0.82(1010) Solve the first equation for x: x = 1010 – y Substitute this into the second equation and solve: 0.92(1010 – y) + 0.70y = 0.82(1010) 929.2 – 0.92y + 0.70y = 828.2 0.22y = 101 **<u>y = 459.1 lbs</u>** Substitute this into the original equation to solve for x: x = 1010 – y **<u>x = 550.9</u>**	x + y = 537 0.92x + 0.70y = 0.88(537) Solve the first equation for x: x = 537 – y. Substitute this into the second equation and solve: 0.92(537 – y) + 0.70y =0 .88(537) 494.04 –.92y + 0.70y = 472.56 0.22y = 21.48 **<u>y = 97.6</u>** Substitute this into the original equation to solve for x: x = 537 – y **<u>x = 439.4</u>**

Now to determine the difference in total profit we need to know the cost for each mixture:
Total Cost = (Total Pounds Bull Meat)($2.07 per pound) + (Total Pounds Lean Trim)($1.82 per pound)

Regular Ground Beef – Using Bull Meat	**Lean Ground Beef – Using Bull Meat**	**Extra Lean Ground Beef – Using Bull Meat**

Cost = 110.2(2.07) + 374.8(1.82) **Total Cost = $910.25**	Cost = 550.9(2.07) + 459.1(1.82) **Total Cost = $1975.92**	Cost = 439.4(2.07) + 97.6(1.82) **Total Cost = $1087.19**

To find the profit at the same price:
Profit = (price per pound)(total pounds) – total cost

Ground Beef: $2.16(485) – $910.25 = **$137.35**
Lean Ground Beef: $2.26(1010) – $1975.92 = **$306.68**
Extra Lean Ground Beef: $2.35(537) – $1087.19 = **$174.76**
Total Profit: **$618.79**

Using Boneless Round the total profit was: $593.71, therefore he makes **<u>$25.08 more profit</u>**, by replacing Boneless Round with Bull Meat.

7. A random sample of 376 customers reveals that 127 bought Extra Lean Ground Beef; how many in the next 500 customers could be expected to purchase Extra Lean Ground Beef.

$$\frac{127}{376} = \frac{z}{500}$$

376z = 127 x 500 = 63,500

z = 63,500/376 = 168.88 = **<u>169 customers</u>**

Week 5

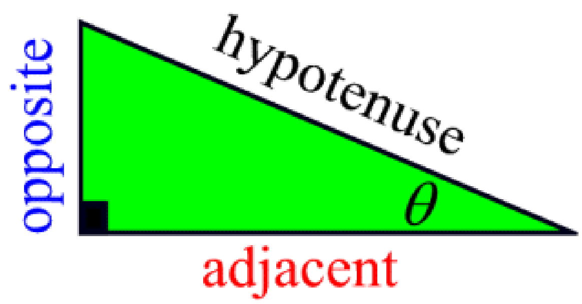
Domain: *Geometry*

Standards:
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Right Triangle Trigonometry

Directions: Study the information below.

In a right triangle, there are actually six possible trigonometric ratios, or functions. A Greek letter (such as theta θ or phi φ) will now be used to represent the angle.



sine of $\theta = \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	cosecant of $\theta = \csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
cosine of $\theta = \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	secant of $\theta = \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
tangent of $\theta = \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	cotangent of $\theta = \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Notice that the three new ratios at the right are reciprocals of the ratios on the left. Applying a little algebra shows the connection between these functions.

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\dfrac{\text{opposite}}{\text{hypotenuse}}} = \frac{1}{\sin\theta}$$

Use the information in the chart to answer the following questions:

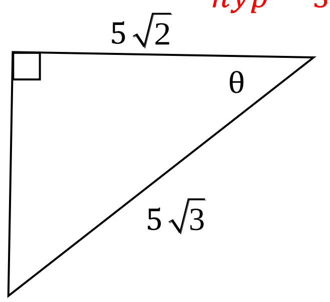
1. If $\sin(\theta) = \frac{4}{5}$, what is the $\csc(\theta)$? Explain your reasoning.
5/4; reciprocal functions

2. How do you find a side length and angle measure in a right triangle?
Pythagorean Theorem and the Triangle Sum Theorem

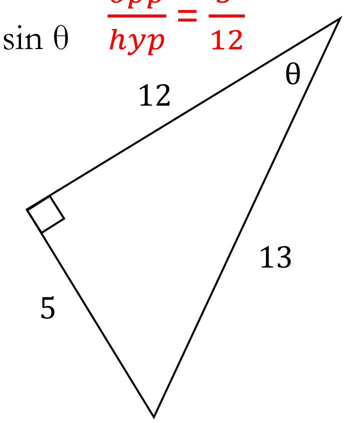
3. Given one trigonometric ratio, how would you find the other five ratios?
Use the Pythagorean Theorem to find the missing side length and then apply those values to the trig ratios.

Use the right triangles to find the given values. Do not simplify.

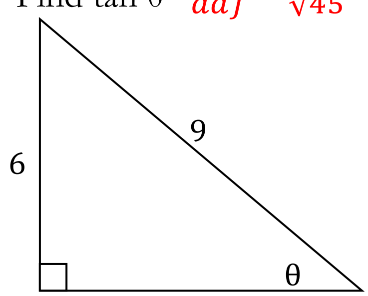
4. Find $\cos\theta$ $\quad \frac{adj}{hyp} = \frac{5\sqrt{2}}{5\sqrt{3}}$



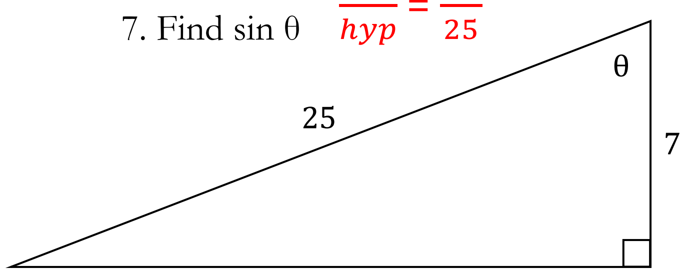
5. Find $\sin\theta$ $\quad \frac{opp}{hyp} = \frac{5}{12}$



6. Find $\tan\theta$ $\quad \frac{opp}{adj} = \frac{6}{\sqrt{45}}$



7. Find $\sin\theta$ $\quad \frac{opp}{hyp} = \frac{24}{25}$



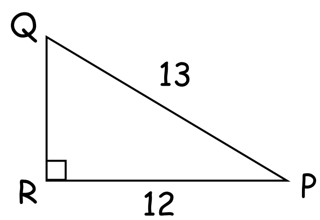
Reciprocal Functions	
$\sin\theta = \dfrac{1}{\csc\theta}$	$\csc\theta = \dfrac{1}{\sin\theta}$
$\cos\theta = \dfrac{1}{\sec\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$
$\tan\theta = \dfrac{1}{\cot\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

Also Important
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$\frac{\sin\theta}{\cos\theta} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{adj} = \tan\theta$$

Find the missing side of the triangle below and give the value of each trigonometric ratio.

1. $\sin(P) =$ 5/13
2. $\sin(Q) =$ 12/13
3. $\cos(P) =$ 12/13
4. $\tan(P) =$ 5/12



5. $\cot(Q) =$ 5/12
6. $\sec(P) =$ 13/12
7. $\csc(P) =$ 13/5
8. $\tan(Q) =$ 12/5

9. Given the cos(θ) = 3/5, find the other five trigonometric ratios.

sin(θ) = 4/5 tan(θ) = 4/3 csc(θ) = 5/3

sec(θ) = 5/4 cot(θ) = 3/4

Week 6

Domain: *Number and Quantity; Expressions and Equations*

Standards:
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. ~~Know that √2 is irrational~~.

Simplifying Radicals

When working with the simplification of radicals you must remember some basic information about ***perfect square*** numbers.

You need to remember:

Perfect Squares
4 = 2 x 2
9 = 3 x 3
16 = 4 x 4
25 = 5 x 5
36 = 6 x 6
49 = 7 x 7
64 = 8 x 8
81 = 9 x 9
100 = 10 x 10

Square roots
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

While there are certainly many more perfect squares, the ones appearing in the charts above are the ones most commonly used.

To simplify means to find another expression with the same value. It does not mean to find a decimal approximation.

1. Find the **largest** perfect square which will divide evenly into the number under your radical sign. This means that when you divide, you get no remainders, no decimals, no fractions.

Reduce: $\sqrt{48}$

the **largest** perfect square that divides evenly into 48 is **16.**

If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

2. Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

$$\sqrt{48} = \sqrt{16 \cdot 3}$$

3. Give each number in the product its own radical sign.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

5. You now have your answer.

$$\sqrt{48} = 4\sqrt{3}$$

Simplify: $3\sqrt{50}$

Don't let the number in front of the radical distract you.
It is simply "along for the ride" and will be multiplied times our final answer.

The largest perfect square dividing evenly into 50 is 25.

$$3\sqrt{50} = 3\sqrt{25 \cdot 2} = 3\sqrt{25}\sqrt{2}$$

Reduce the "perfect" radical and multiply times the 3 (who is "along for the ride")

$$3\sqrt{25}\sqrt{2} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$$

Simplify the following.

a) $\sqrt{28} =$

$2\sqrt{7}$

b) $\sqrt{50} =$

$5\sqrt{2}$

c) $\sqrt{45} =$

$3\sqrt{5}$

d) $\sqrt{98} =$

$7\sqrt{2}$

e) $\sqrt{48} =$

$4\sqrt{3}$

f) $\sqrt{300} =$

$10\sqrt{3}$

g) $\sqrt{150} =$

$5\sqrt{6}$

h) $\sqrt{80} =$

$4\sqrt{5}$

Reduce to lowest terms.

a) $\frac{\sqrt{20}}{2}$

$\sqrt{5}$

b) $\frac{\sqrt{72}}{3}$

$2\sqrt{2}$

c) $\frac{\sqrt{22}}{2}$

already in lowest terms

Simplify the following expressions.

$$\frac{6 \pm 2\sqrt{45}}{6}$$

$1 \pm \sqrt{5}$

$$\frac{10 \pm 3\sqrt{12}}{4}$$

$\frac{5 \pm 3\sqrt{2}}{2}$

$$\frac{5 \pm \sqrt{75}}{10}$$

$\frac{1 \pm \sqrt{3}}{2}$

$$\frac{3 \pm \sqrt{121}}{4}$$

$\frac{7}{2}; -2$

$$\frac{2 \pm 4\sqrt{7}}{6}$$

$\frac{1 \pm 2\sqrt{7}}{3}$

$$\frac{2 \pm 6\sqrt{14}}{4}$$

$\frac{1 \pm 3\sqrt{14}}{2}$

$$\frac{8 \pm 4\sqrt{12}}{16}$$

$\frac{1 \pm \sqrt{3}}{2}$

$$\frac{1 \pm \sqrt{32}}{4}$$

$\frac{1 \pm 4\sqrt{2}}{2}$

$$\frac{4 \pm \sqrt{28}}{6}$$

$\frac{2 \pm \sqrt{7}}{3}$

$$\frac{7 \pm \sqrt{98}}{7}$$

$1 \pm \sqrt{2}$

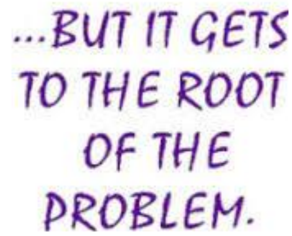
$$\frac{9 \pm \sqrt{54}}{6}$$

$\frac{3 \pm \sqrt{6}}{2}$

$$\frac{12 \pm \sqrt{200}}{10}$$

$\frac{6 \pm 5\sqrt{2}}{2}$

Week 7

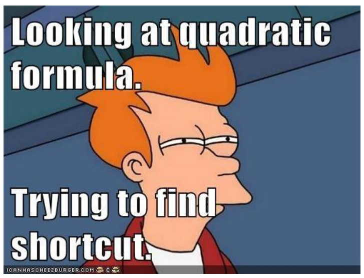


Domain: *Algebra*

Standards:

A.REI.4 Solve quadratic equations in one variable.

A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.



The quadratic formula can be used to solve quadratic equations that cannot be solved by factoring over the integers. The quadratic formula can be very useful in solving problems. Thus, it should be practiced enough to develop accuracy in using it and to allow you to commit the formula to memory.

Quadratic Expression	Quadratic Equation	Quadratic Formula
$ax^2 + bx + c$	$ax^2 + bx + c = 0$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Using the Quadratic Formula:

1) Equation must be in quadratic form.
2) Identify a, b, and c.
3) Input values into the formula and simplify.

$2x^2 - 3x - 10 = 0$

a =2, b=-3, c =-10

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(2)(-10)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9+80}}{4} = \frac{3 \pm \sqrt{89}}{4}$$

Shortcut - To limit mathematical errors and shorten steps when using the quadratic formula:

1) Identify a, b, and c.
2) Find $b^2 - 4ac$.
3) Input values into the formula and simpliy.

$2x^2 - 3x - 10 = 0$

a =2, b=-3, c =-10

$b^2 - 4ac = (-3)^2 - 4(2)(-10) = 89$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{89}}{2(2)} = \frac{3 \pm \sqrt{89}}{4}$$

Solutions: $x = \frac{3 + \sqrt{89}}{4}$ and $x = \frac{3 - \sqrt{89}}{4}$

Approximate solutions for this equation are 3.1085 and -1.6085.

Use the quadratic formula to solve each of the following quadratic equations. Approximate your solutions to two decimal places.

Quadratic Equations	**Identify a, b, c**	**Find b^2- 4ac**	**Substitute into** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x^2 - 7x + 10 = 0$	1, -7, 10	9	x = 2 and x = 5
$x^2 - 14x + 45 = 0$	1, -14, 45	16	x = 5 and x = 9
$0.25x^2 - 0.25x - 10.5 = 0$	.25, -.25, .10.5	10.5625	x = -.38 and x = .44
$4x^2 + 8x = 77$	4, 8, -77	1296	x = -88 and x = 56
$x^2 - 11x + 30.25 = 0$	1,-11, 30.25	0	x = 5.5
$3 = 16p - 20p^2$	-20, 16, -3	16	x = 120 and x = 200
$x^2 = 15 + 2x$	1, 2, -15	64	x = -3 and x = 5
$x^2 - 10x + 35 = 7x - 35$	1, -17, 70	9	x = 7 and x = 10
$2x^2 + 5 + 20x = 20 - 8x^2 + x$	10, 19, -15	961	x = -250 and x = 60

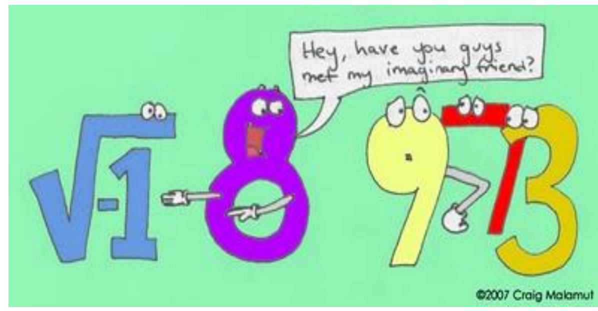
Algebra II – Unit 1 Preview

Domain: *Number and Quantity*, Algebra

Standards:
N.CN.1 - Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
N.CN.7 – Solve quadratic equations with real coefficients that have complex solutions.
A.CED.1– Create equations that describe numbers or relationships.
A.SSE.1 – Interpret the structure of expressions.

Imaginary Number

An Imaginary Number, when squared, gives a negative result: $i^2 = -1$; The "unit" Imaginary Number (the equivalent of **1** for Real Numbers) is $\sqrt{-1}$ (the square root of minus one).



If ***a*** is a positive real number, then the principal square root of negative ***a*** is the imaginary number $i\sqrt{a}$; that is, $\sqrt{-a} = i\sqrt{a}$.

Example 1: $\sqrt{-7} = \sqrt{7} \cdot \sqrt{-1} = i\sqrt{7}$

Example 2: $\sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i$

Example 3: $\sqrt{-99} = \sqrt{9} \cdot \sqrt{11} \cdot \sqrt{-1} = 3 \cdot i \cdot \sqrt{11} = 3i\sqrt{11}$

Simplify each expression.

1. $\sqrt{-16}$

4i

2. $\sqrt{-28}$

$2i\sqrt{7}$

3. $\sqrt{-5}$

$i\sqrt{5}$

4. $\sqrt{-10} \bullet \sqrt{-2}$

$2\sqrt{5}$

5. $\dfrac{\sqrt{-50}}{\sqrt{-10}}$

$\sqrt{5}$

6. $\sqrt{64-100}$

6i

Complex Number

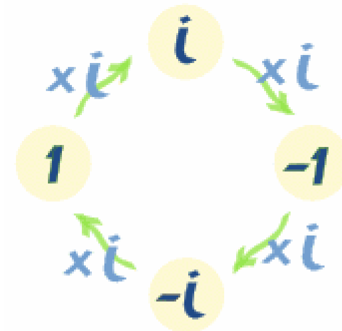
A *complex number* is a number in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The number a is the real part of $a + bi$ and bi is the imaginary part. Any solution should be written in this form.

So, a Complex Number has a real part and an imaginary part.

But either part can be **0**, so all Real Numbers and Imaginary Numbers are also Complex Numbers.

Complete the table.

Complex Number	Real Part	Imaginary Part
3 + 2i	3	2
5	5	**0**
-6i	**0**	-6
1 + 7i	**1**	**7**
5.2i	**0**	**5.2**
4	**4**	**0**
2 - 5i	**2**	**-5**
2 + πi	**2**	**π**
8.2 – 5.5i	**8.2**	**-5.5**
$\sqrt{2} + \frac{i}{2}$	**$\sqrt{2}$**	**1/2**
$\sqrt{-1}$	**0**	**1**



Patterns of *i*

The Unit Imaginary Number, ***i***, has an interesting property. It "cycles" through 4 different values each time you multiply. To determine the value of ***i*** to any power just find the largest multiple of 4 and ***i*** to that multiple of 4 =1 then the remainder is the exponent of ***i*** that is evaluated and multiplied by 1.

Example: $i^{347} = i^{344} i^{3} = (1)(-i) = -i$.

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$$i^n = \begin{cases} 1 & \text{if the remainder is 0} \\ i & \text{if the remainder is 1} \\ -1 & \text{if the remainder is 2} \\ -i & \text{if the remainder is 3} \end{cases}$$

Evaluate:

1. i^{30} **-1**

2. i^{210} **-1**

3. $3i^{7}$ **-3i**

4. $-25i^{25}$ **-25i**

5. $i + i^{3}$ **0**

6. $\mathrm{i}^{19} - i^{16}$ **-1-1i**

7. $3\mathrm{i}^{4} + 5i^{6}$ **-2**

8. $\mathrm{i}^{37} \div i^{35}$ **-1**

9. $\mathrm{i}^{102} \bullet i^{-98}$ **1**

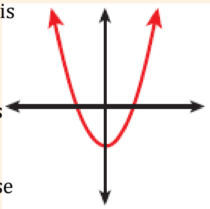
10. $\left(i^{2}\right)^{3}$ **-1**

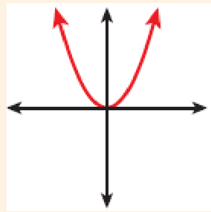
Discriminant

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

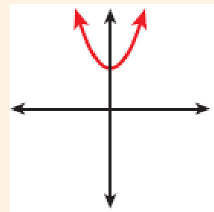
$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
two distinct real solutions	one distinct real solution	two distinct nonreal complex solutions

If b^2-4ac is a perfect square, then solutions are rational. Otherwise they are irrational.





Produces two equal rational roots



Calculate the discriminant to determine the number and nature of the solutions of the following quadratic equation:

Quadratic	Identify a, b, c	Discriminant (b^2-4ac)	Number of Solutions	Nature of Solutions
$y = x^2 + 4$	**1, 0, 4**	**-16**	**2**	**non-real, complex**
$y = x^2$	**1, 0, 0**	**0**	**1**	**real, rational**
$y = -x^2 - 5x - 50$	**-1, -5, -50**	**-175**	**2**	**non-real, complex**
$-6x^2 - 6 = -7x - 9$	**-6, 7, 3**	**121**	**2**	**Real, rational**
$4k^2 + 5k + 4 = -3k$	**4, 8, 4**	**0**	**1**	**Real, rational**
$-7n^2 + 16n = 8n$	**-7, 8, 0**	**64**	**2**	**Real, rational**
$2x^2 = 10x + 5$	**2, -10, -5**	**140**	**2**	**Real, irrational**
$-9b^2 = -8b + 8$	**-9, 8, -8**	**-224**	**2**	**Non-real, complex**
$5b^2 + b - 2 = 0$	**5, 1, -2**	**-41**	**2**	**Non-real, complex**
$-2x^2 - x - 1 = 0$	**-2, -1, -1**	**-7**	**2**	**Non-real, complex**
$-4m^2 - 4m + 5 = 0$	**-4, -4, 5**	**96**	**2**	**Real, irrational**
$y = -x^2 - 5x + 50$	**-1, -5, 50**	**225**	**2**	**Real, rational**
$y = -x^2 - 4x - 4$	**-1, -4, -4**	**0**	**1**	**Real, rational**
$8b^2 - 6b + 3 = 5b^2$	**3, -6, 3**	**0**	**1**	**Real, rational**
$9n^2 - 3n - 8 = -10$	**9, -3, 2**	**-63**	**2**	**Non-real, complex**

Critical Thinking Questions:

a) Write a quadratic equation that has two imaginary solutions.

Answers vary.

b) In $y=2x^2-4x+k$, find k that will produce two equal real roots. Explain how you determined your answer.

k = 2

Explanation:
(the discriminant must equal 0 in order to produce equal roots)
$a=2$, $b= -4$, $c=?$
$b^2 - 4ac = 0$
$(-4)^2 - 4(2)c = 0$
$16 - 8c = 0$
$-8c = -16$
$c = 2$

c) In your own words explain why a quadratic equation can't have one imaginary solution.

Sample Response:
An imaginary solution means that the parabola never touches the x-axis. Whenever a parabola touches the x-axis, it is what we call a solution. Whenever the parabola does not touch the x-axis, it produces two imaginary solutions. There cannot be only one imaginary solution because the power of the quadratic equation is 2. Because this power is 2, there must be 2 solutions. Even when the parabola only touches the x-axis once, it still produces 2 solutions, even though they are the same.